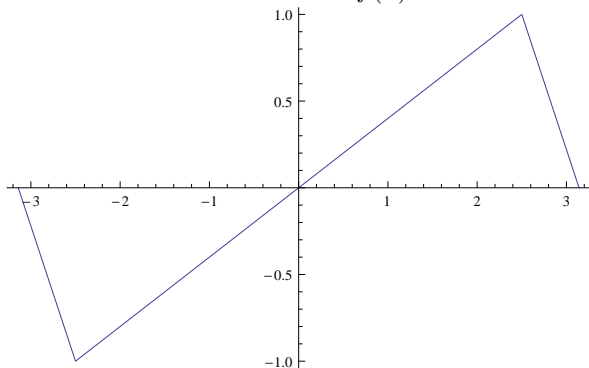


Math Homework #1

- Let $0 < a < \pi$ be some number, and define a 2π -periodic odd function $f(x)$ by its values on the interval $0 \leq x \leq \pi$:

$$f(x) = \begin{cases} x/a & \text{if } 0 \leq x \leq a \\ 1 - \frac{x-a}{\pi-a} & \text{if } a < x \leq \pi \end{cases}$$

Calculate a Fourier series for $f(x)$. Does this series fail to converge at any point (why or why not)?



- Suppose that the part of the graph of $f(x)$ above represents the initial temperature (in some units) of a metal rod. The temperature of the rod is governed by the heat equation $u_t = au_{xx}$ for some constant a , and the ends of the rod are kept at a constant temperature of 0. Give a Fourier series solution for the temperature of the rod as a function of time.
- (Exercise 3 of section 2.2 in Folland's book) Calculate a Fourier series for the function $f(x) = |\sin(x)|$. Evaluate the series you obtain at $x = 0$ and $x = \pi/2$ to calculate the values of the following sums:

(a) $\sum_{k=1}^{\infty} \frac{1}{4n^2-1} = ?$

(b) $\sum_{k=1}^{\infty} \frac{(-1)^n}{4n^2-1} = ?$

- Suppose two packs of playing cards are shuffled together, and a five card poker hand is dealt. Calculate the probabilities of the following hands:
 - Five of a kind
 - Four of a kind (or more)
 - Three of a kind (or more)
 - A pair (or more)

By "or more", I mean for example that you don't have to subtract of the chance of five of a kind when computing four of a kind.

- (Problem 3.1 from *A First Course in Probability* by Sheldon Ross) Two fair dice are rolled. What is the conditional probability that at least one lands on 6 given that the dice land on different numbers?
- In a certain town in the Wild West, 35% of the residents are farmers, 45% are gold miners, and 20% are outlaws. Many of the residents have wooden teeth, in fact this is the case for 15% of the farmers, 30% of the gold miners, and 70% of the outlaws. Given that a citizen has wooden teeth, what is the conditional probability that he is an outlaw?